A COMPUTER ASSISTED METHOD FOR DETERMINATION OF INFLATED DIMENSIONS OF PNEUMATIC TYRES

by

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Abstract

When a pneumatic tyre is fitted on a rim and inflated its shape and dimensions change until the tension in the carcass cord reaches equilibrium with the pressure due to inflation. The designer should be able to predict these changes accurately so that they could be minimised or more important, avoid dimensional changes from mould to service conditions. The paper outlines a computer assisted method for precise computation of inflated tyre dimensions.

Background

When a pneumatic tyre (bias ply) is fitted on a rim and inflated its dimensions change significantly on account of two physical factors:

1. to accommodate the elongation in the reinforcing material
2. to reach equilibrium between the internal pressure and the tension in the cord of reinforcing material.

The designer should be able to predict these changes accurately and precisely so that the required dimensions can be met, and more important, to keep the changes to a minimum since any change from moulded dimensions will cause unwanted stresses in rubber parts of the tyre which would affect its durability.

The methods used to calculate and predict dimensional changes are largely empirical and are based on practical experience. Since these methods are not precise designers have considerable difficulty in meeting standards and the cost of trial and error moulds is very high.

The method described below will enable a designer to predict dimensions precisely and easily through the use of a computer.

1.0 Introduction

A cross section of a bias ply tyre fitted on a rim is shown in Fig. 1. The reinforcing structure (carcass) determines the strength, shape and dimensions of the tyre.

The carcass consists of several layers of weftless fabric so that each layer will consist of a series of cord neatly arranged side by side and insulated from each other with rubber compound.

The ends of each cord are anchored to the bead wire and set at a specified angle to the meridian (Fig. 3). This arrangement allows each cord to take up a geometrical position so that it takes only a tension to balance the internal pressure due to inflation. As will be seen later the angle \( \beta \) is a primary factor determining the dimensions of a tyre.

For purposes of calculation of dimensions the carcass will be assumed to be a thin shell at the inner contour (Fig. 2). Once the geometry of this thin shell is determined the external dimensions of the tyre are determined by adding the thickness of material.

2.0 Theoretical considerations

Let us go back to Fig. 1.

The bead area is held firmly against the rim flange and hence the geometry of this area is the same as that of the moulded tyre. The area above the bead is flexible and will be displaced to take up an equilibrium position. Hence a hinge point \( H \) exists above which the carcass will be displaced and below which it is fixed. The hinge point is not clearly defined, however, since the displacement in this region is very small any error in its location will not affect the precision of calculation, and its location as in Fig. 1 is convenient.

Fig. 2 shows the inner contour of a moulded tyre and in broken line its position after inflation. The arc length \( S \)

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increases due to elongation of cord. In practice this leads to growth of a tyre, and a mean growth factor $G$ can be defined as

$$ G = \frac{S_i - S_m}{S_m} \cdot 100 $$

Where suffixes $i$ and $m$ stand for inflated and moulded dimensions respectively.

The shape of the inner contour is dependent on the cord angle $\beta_k$ and the radius of curvature $\rho$ will gradually decrease as we move from C to H.

The Soviet Scientist V.L. Biderman (1) has developed the following relation for the radius of curvature:

$$ \rho = R \frac{1 - \lambda^2}{\lambda} \cdot \cos \beta_k \sqrt{1 - \lambda^2 \sin^2 \beta_k} \frac{\sin^2 \beta_k}{2 - (3\lambda^2 - \lambda^2) \sin^2 \beta_k} $$

(1)

Where $\rho$ = radius of curvature at a point $r$ from the axis of rotation, and

$$ \lambda = \frac{r}{R} \quad \lambda_0 = \frac{r_0}{R} $$

$\beta_k$ = cord angle (see Fig. 3)

Equation (1) cannot be used directly to calculate since only the position of the hinge point H is known, and $R$, $r_0$ are not known.

Hence the foregoing procedure has been developed and implemented at the Sri Lanka Tyre Corporation.

3.0 Calculation of radii of curvature

Let's divide the portion CP into seven segments as shown in Fig. 4. Each segment having an angle of $D = \pi / 14$. The radius of curvature at C is $\beta_k$ and that at $P$ is $\beta$. A segment is presumed to have the same radius of curvature and those of the seven segments 1,2,$\ldots$,7 are $\beta_k, \beta_1, \beta_2, \ldots, \beta_6$ respectively.

For given values of $R$ and $r$, $\beta_k$ can be calculated by substituting in equation (1).

Now

$$ \Delta R_1 = \beta_k (1 - \cos D) $$

and

$$ \gamma_1 = R - \Delta R_1 $$

Hence $\beta_1$ can be calculated by substituting the value of $\lambda$ in equation (1).

and

$$ \Delta R_2 = \beta_1 \cos D - \beta_1 \cos 2D $$

$$ \gamma_2 = \gamma_1 - \Delta R_2 $$

Hence $\beta_2$ can be calculated by substituting in equation (1).

In Similar fashion $\beta_3, \ldots, \beta_7$ can be calculated.

The calculated curve has a small error since the mean radius of a segment has not been used. This error can be minimised by increasing the number of segments.

After calculating the radii of curvature $\beta_k, \beta_1, \beta_2, \ldots, \beta_7$ $W$ can be calculated from

$$ W = (\beta_k - \beta_1) \sin D + (\beta_1 - \beta_2) \sin 2D + \ldots + (\beta_6 - \beta_7) \sin 6D + \beta_7 $$

The arc length $\Delta S_n$ for each segment is given by

$$ \Delta S_n = \frac{\rho_n + \rho_{n-1}}{2} \cdot D $$

Hence the arc length from C to $P$ is given by

$$ S_{CP} = (\beta_k + \beta_1 + \beta_2 + \ldots + \beta_6 + \beta_7) \cdot D $$

The calculation of the portion between P and hinge point H is as given below.

Since the arc length is smaller than CP it is sufficient to divide the portion into 3 segments. The first two segments can be of equal angle, and that of the third segment will be calculated.

Now approximately

$$ \sin \beta = \frac{r_0 - \gamma_1}{\rho_7} $$

Hence $\gamma$ and $\gamma_1$ can be found. Let's divide $P$H into segments so that the angle of first two segment is $\gamma_1$ and that of third is $\delta$.

$$ \Delta \beta_8 = \beta_7 \sin \gamma_1/3 $$

and

$$ \gamma_8 = \gamma_0 - \Delta \beta_8 $$

hence $\beta_8$ can be calculated

$$ \Delta \beta_9 = (\beta_8 - \beta_7) \sin \gamma_1/3 + \beta_8 \sin \frac{2\gamma}{3} $$

$$ \beta_9 = \beta_0 - \Delta \beta_9 $$

hence $\beta_9$ can be calculated

Now it is necessary to calculate angle $\delta$

Now $\beta_9 \sin (\delta + \frac{2\gamma}{3}) = (r_0 - r_1) - (\beta_8 - \beta_7)$

$$ \sin \frac{2\gamma}{3} - (\beta_7 - \beta_8) \sin \gamma_1/3 $$

$\beta_{10}$ can be calculated by substituting $\gamma_1/3$ in equation (1), and then $\delta$ can be calculated.

Now the distance is calculated from the equation.

$$ x = \beta_1 - (\beta_1 - \beta_7) \cos \gamma_1/3 - (\beta_8 - \beta_9) \cos \frac{2\gamma}{3} - \beta_9 \cos (\frac{2\gamma}{3} + \delta) $$

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START

Input Min & Max βk
Min & Max R
Thickness, Tc & Ts
r₁, r₂, d₁n, s₁n

Compute cos D cos 2D... cos 6D
Compute sin D sin 2D... sin 6D
Compute initial value of f₁ (r₁ max)

Select value of β₁
Min & Max values of βk as input & step as programmed

Compute cos β₁ & sin β₁

Select value of R
Min & Max R as input step as programmed

Select value of s₁n
Max value as calculated above. Min value = r₁

Compute P₁
Compute ΔR₁ r₁ and then P₁
Compute ΔR₂ r₂ and then P₂
..............
Compute.............. P₁₀
Compute W₁ X₁ and then d₁

If d₁ > d₁n
No
Yes

Compute OD WIDTH, arc length and then G

Print β₁, OD, width and G

Is max value of R reached?
No
Yes

Is max value of β₁ reached?
No
Yes

Await instructions on further computations.
For convenience the portion of arc from \( P \) to hinge point \( H \) has been divided into 3 segments only. If more precision is desired it can be divided into any number of segments and the angle in the last segment can be calculated the same way as \( \delta \) was calculated.

The author finds that practically no error is introduced if the portion is assumed to consist of a radius \( P \) at mid point between \( P \) and hinge point \( H \). \( P \) is calculated by substituting \( r = \frac{r_0 + r_1}{2} \) in equation (1) and angle calculated from \[ \sin \theta = \frac{r_0 - r_1}{r_1} \]

It also considerably reduces the computation time for the machine, bearing in mind that this will have to be repeated several times.

Once \( x \) is found the distance \( d \) of the calculated contour can be found from the relation

\[ d = \omega - x \]

Calculation of the arc length \( \phi \)

This arc length is given by

\[ \phi = \frac{\pi}{3} \left( \frac{P_1 + P_0}{2} \right) + \frac{\pi}{3} \left( \frac{P_2 + P_0}{2} \right) + \pi \left( \frac{P_2 + P_0}{2} \right) \]

Where \( P_0 \) has to be calculated by substituting \( r = r_0 \) in equation (1).

The total arc length \( S_t \) is given by

\[ S_t = Scp + Sph \]

and the growth per cent is given by

\[ G = \frac{S_t - Sm}{Sm} \times 100 \]

Once \( d \) is found we can repeat calculations until \( d = d_{in} \) and then proceed to calculate \( G \)

Where \( d_{in} \) is the input value of \( d \).

Since it is not practical with a computer to repeat calculation until \( d = d_{in} \) the following procedure can be adopted. The initial value of \( r_0 \) is selected greater than \( r_0 \) of the required contour so that \( d \) is less than \( d_{in} \). The calculation could then be repeated until is greater than \( d_{in} \).

Initial data required
1. Arc length from crown to hinge point \( Sm \)
2. Distance \( r_0 \) and \( d \) (Note should be half the width of the recommended rim irrespective of its value in the drawing)
3. Thicknesses \( t_c \) and \( t_s \)
4. Approximate tyre overall diameter (OD)

Determination of growth factor \( G \)

The growth factor is determined by making measurements on available sizes of tyres.

The tyres are mounted on specified rims, inflated to specified pressures and allowed to rest at ambient temperature for 24 hours. The inflation pressure is re-adjusted and the overall circumference and the tyre section width are measured. The diameter is calculated from the circumference.

The tyre will then undergo the drum test, after which it is cut to measure the cord angle \( \phi \) and thickness at crown \( t_c \) and thickness at sidewell \( t_s \).

The data thus obtained can be compared with the computer calculated values and the correlation between inflated dimensions and the growth factor can be obtained for each type of tyre.

Some of the data obtained and the determined growth factor are given in Table 1.

Implementation at SLTC

The developed computer programme has been in use at SLTC since 1988 and several new designs and sizes introduced are such computer assisted designs.

The computer predicted dimensions, and the actual measurements taken after manufacture are given in Table 2. Since the prediction of dimensions has been precise the design of the mould has been precise and this has contributed to improved performance of these tyres.
<table>
<thead>
<tr>
<th>Tyre size</th>
<th>Measured data</th>
<th></th>
<th>Fitting computer predicted dimensions</th>
<th>Fitting growth factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Overall</td>
<td>Section</td>
<td>Angle</td>
</tr>
<tr>
<td>7.50-16(12 PR)</td>
<td>806</td>
<td>216</td>
<td>56°</td>
<td></td>
</tr>
<tr>
<td>6.00-14(6 PR)</td>
<td>667</td>
<td>175</td>
<td>53°</td>
<td></td>
</tr>
<tr>
<td>6.00-12(4 PR)</td>
<td>575</td>
<td>150</td>
<td>52°</td>
<td></td>
</tr>
<tr>
<td>5.90-13(4 PR)</td>
<td>613</td>
<td>159</td>
<td>54°</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1**

<table>
<thead>
<tr>
<th>Tyre size</th>
<th>Computer predicted dimensions</th>
<th>Actual measurement after manufacture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dia</td>
<td>width</td>
</tr>
<tr>
<td>7.50-15</td>
<td>775</td>
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<td>140</td>
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<tr>
<td>5.00-12</td>
<td>570</td>
<td>138</td>
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<tr>
<td>5.50-13</td>
<td>623</td>
<td>157</td>
</tr>
</tbody>
</table>

**Table 2**

Reference